Qualifying Exam Syllabus

Hannah Friedman

at 2 pm on December 2, 2024 in Evans 939

Committee: Mark Haiman (Chair), Bernd Sturmfels (Advisor), Serkan Hoşten (Co-adivsor), David Eisenbud

1 Major Topic: Combinatorics (Algebra)

References: Richard Stanley, *Enumerative Combinatorics*, Chapters 1-3,5,7; William Fulton, *Young Tableaux*, Chpaters 1-9

- Enumeration: inversions, partitions, The Twelvefold Way, Stirling numbers, ordinary and exponential generating functions, Inclusion-Exclusion, involutions, posets operations, distributive lattices, incidence algebras, Möbius inversion, Lagrange Inversion Formula, Matrix-tree theorem
- Symmetric Functions: bases for the space of symmetric functions, Hall inner product, Kostka numbers, Pieri rule, Littlewood-Richardson rule, Jacobi-Trudi Identity
- Calculus of Tableaux: hook formula, RSK, Viennot's construction, increasing and decreasing subsequences
- Applications to Representation Theory: Specht modules, branching rule, Schur modules, Plücker embedding, Schubert Calculus on Grassmannians

2 Major Topic: Algebraic Statistics (Applied Math)

References: Seth Sullivant, *Algebraic Statistics*, Chapters 1-8, 13-15

- Exponential Families: regular exponential families, discrete exponential families, gaussian exponential families, algebraic exponential families
- Likelihood Inference: statistical models, parameter estimation, likelihood geometry, ML degree, Varchenko's Theorem
- Conditional Independence: marginal independence, intersection axioms, conditional independence models, primary decomposition of CI ideals
- Graphical and Phylogenetic Models: parametrization, Hammersley-Clifford Theorem, mixture models, hidden variable graphical models, group-based phylogenetic models, the general Markov model, the Allman-Rhodes-Draisma-Kuttler Theorem

3 Minor Topic: Commutative Algebra (Algebra)

References: David Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Chapters 2-4, 8-10, 12-13, 15

- Rings and Modules: Hilbert Basis Theorem, Hilbert functions, Hilbert Polynomials, localization, tensor products, associated primes, prime avoidance, primary decomposition, Nakayama's Lemma, the Nullstellensatz
- Gröbner Bases: initial ideals, monomial ideals, Buchberger's algorithm, elimination, ideal membership, syzygies
- Dimension Theory: Krull dimension, system of parameters, principal ideal theorem, regular local rings, Hilbert-Samual functions, integral extensions, lying over, going up and going down, Noether normalization